A Condition-Based Maintenance Policy Based On A Probabilistic Meta-Model In The Case Of Chloride-Induced Corrosion

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ABSTRACT: Maintenance and management policies are usually focused on minimizing the life-cycle cost only. Therefore the optimal solution in this context does not necessarily result in a satisfactory long-term structural performance. In this paper, we will present an approach for modeling the degradation of structures and infrastructures for maintenance purposes. The degradation is modeled using probabilistic data-driven state dependent stochastic processes, hereafter called meta-model. This work implements this degradation model into a maintenance framework and carries out two numerical examples in order to show the applicability of our meta-model in a maintenance and management optimization context. This paves the road for future work on meta-model updating and maintenance optimization by considering a multi-objective optimization policies.

1. INTRODUCTION

Growth and achievement in society are ensured by the quality of operation and performance of the infrastructure: roads, power lines, ports, dams, bridges, etc...

Previous studies have always searched for methods to measure and monitor the performance of structures in order to avoid excessive degradation leading to unacceptable risky situations. It is highly important to keep track of the evolution of degradation in materials and predict its level to avoid failure by maintaining the structure in the "safe" zone.

In civil engineering, inspections can be classed into two categories: destructive and non-destructive techniques (DT and NDT respectively). As the name indicates, in order to carry out a DT inspection the structure is harmed. It's up to the inspection consultant to decide whether the inspection will alter the global performance of the structure or just give a bad visual sensation (not less important than the performance in some cases). On the other hand, NDT do not affect the performance of the structure. From a structural and managerial point of view, it is clearly preferable to use NDT over DT, but NDT measurements could be costly and in some cases, results are less reliable than the obtained by DT.

Corrosion of reinforcement steel is known to be one of the major causes of deterioration of reinforced concrete (RC) structures (Bastidas-Arteaga *et al.*, 2012, Bastidas-Arteaga *et al.*, 2011). In this paper, we focus on the case of chloride-induced corrosion in submerged reinforced concrete.

The purpose of this paper is to propose a maintenance and management model for this pathology. Most existing maintenance systems focus on life-cycle cost minimization only. Therefore, the obtained solution does not necessarily result in satisfactory long-term structural performance (Frangopol & Liu, 2007). Moreover, complex multi-parametric degradation models were developed for representing the main trends, but not for (i) uncertainty propagation, and (ii) updating from NDT results that are generally not directly linked to the model. Meta-models have been shown to be efficient in the case where the degradation can be modeled with a single Markov matrix (O'Connor *et al.,* 2013, Bastidas-Arteaga *et al.,* 2012). This paper addresses the case of a complete three-step problem.

The aim of the proposed approach is to estimate the life-cycle cost and the condition index for different proposed solutions. Therefore, on one hand we calculate a "condition index" through which overall structural condition is represented. On the other hand, we calculate life-cycle cost from a set of possible maintenance strategies, inspections, and design costs.

Section 2, introduces the degradation process. Section 3, shows the development of the meta-model. Section 4 describes the maintenance actions and performance indexes that can be considered. Section 5 illustrates the proposed approach with numerical examples of different maintenances policies. And finally, the conclusions and perspectives of this work are drawn in section 6.

2. PROBLEM STATEMENT

Chloride ingress into RC structures leads to serviceability and safety losses. Deterioration modelling allows estimating the effects of chloride ingress with regard to serviceability or ultimate limit states. Ultimate limit states are highly dependent on both, geometrical characteristics (cross-sectional dimensions, span length, etc.) and loading (dead, live, seismic, etc.). Therefore, to generalize the results, this work focuses on a serviceability limit state related to the time to corrosion damage of the concrete cover (severe cracking or spalling). Corrosion-induced cover cracking and damage occurs on the concrete surface above and parallel to the rebars. The time to corrosion damage, (severe cracking or spalling), is thus obtained as the sum of three stages (Figure 1): (i) corrosion initiation; (ii) crack initiation (time to first cracking - hairline crack of 0.05 mm width), and; (iii) crack propagation (time for crack to develop from crack initiation to a limit crack width, w_{lim}).

The corrosion initiation phase is controlled by the diffusion of chlorides into concrete. When the chloride concentration at the surface of the steel (cover depth) exceeds a threshold concentration, there is steel depassivation followed by corrosion initiation.

The crack initiation phase is dominated by the chemical reaction of corrosion generating corrosion products. These corrosion products or rust slowly fill the pores surrounding the reinforcement steel creating an internal pressure. When the internal pressure exceeds the concrete resistance, hairline cracks appear.

Finally, the propagation phase lies in the continuous accumulation of rust generating more internal tensile stress, resulting in an excessive cracking of the concrete cover.

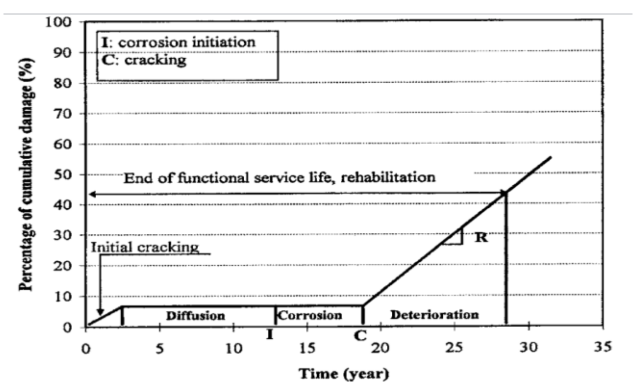

*Figure 1. Degradation by corrosion of reinforced concrete (*50 mm *cover) by (Youping Liu, 1996)*

The proposed probabilistic degradation meta-model is based on a small number of "physical" indicators (two per phase) chosen in a way to be accessible through NDT inspections and to provide a truthful degradation level. For each phase, we selected the following physical indicators.

- 1st phase: Chloride concentration at the surface of the steel [Cl^-] and the concrete pH.
- 2nd phase: Internal tensile stress and corrosion current density.
- 3rd phase: Crack width and corrosion current density (Li *et al.*, 2006).

3. META-MODELING

This section shows the formulation of the degradation meta-model. Since this is a three-phased model, we will start by laying down the common ideas behind the model. Then we proceed to explicit the mathematical equations.

For each phase, we propose to define a bivariate process written $(\rho_t, \theta_t)_{\forall t \geq 0}$ as a state dependent stochastic process similar to the one introduced in Zouch *et al*. (2011): ρ_t describing a condition state and θ_t a potential of its evolution (observed), both being dependent. The evolution of degradation over a period of time is given by positive increments for the degradation processes respectively ($\Delta\rho$, $\Delta\theta$) which are continuous random variables. We assume that the degradation increments in a given time interval τ are random variables which are a function of the present degradation state(ρ_t, θ_t). The degradation process is therefore assumed to be a Markov process. A suitable candidate for the distribution laws of each increment ($\Delta\rho$, $\Delta\theta$) is the gamma distribution (Van Noortwijk, 2009) defined by two parameters *(*α and β where: α is the shape parameter and β is the scale parameter). In the described bivariate state-dependent model, we will consider that only the shape parameter is a function of the current state $(\rho_t, \theta_t)$and τ, but independent of time.

The construction of the dependence of the two sub processes is motivated by mechanical expert judgments; there is a cause-effect relationship between the two processes. For instance, for the second and third phases, the corrosion current density is the *cause*, and the width of the crack and internal stress are the *effect*. When corrosion current density increases, the tensile stresses on concrete also increase accelerating concrete cracking initiation and propagation. At the same time, the presence of cracks induces more oxygen and humidity near the corrosion cell resulting in an increasing of the corrosion current density (mutual dependencies).

The correlation is modeled in terms of mutual acceleration effects directly in each of the shape parameters of the gamma distributions. This model is sequential in the sense that for each phase, we first seek to characterize the evolution in terms of the *causal* process then doing so for the respective *effect* process.

To simplify the identification step, we consider that the state dependence is exclusively governed by the shape functions; the scale functions β_θ and β_ρ are considered constant throughout the life cycle. Therefore, we have to model and identify the shape functions α_θ and α_ρ which are respectively, in the case of the gamma distribution, proportional to the expected value of the increments for θ and ρ on the time interval τ.

The choices for each shape function is motivated by the evolution of its respective physical parameter over time. Figure 1 shows simulations of the whole process.

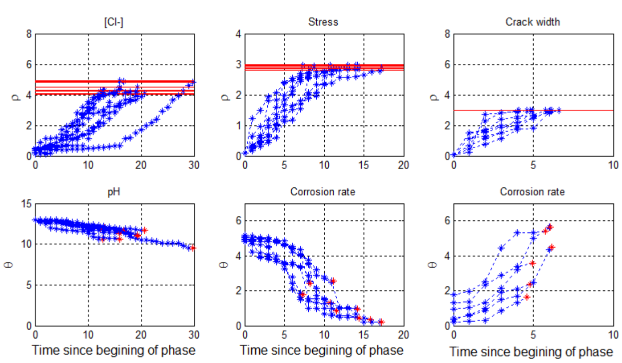

Figure 2. Degradation simulations

A detailed example of the construction and performance analysis of a meta- model applied to the third phase can be found in El Hajj *et al.*

(2014, a). Further studies on the estimation process in the case of censored data can be found in El Hajj *et al*. (2014, b).

3.1. First phase

The corrosion initiation phase is then characterized by two parameters:

- $(\rho_{1,t})_{\forall t \geq 0}$ represents the concentration of chloride at the surface of the steel [Cl^-].
- $(\theta_{1,t})_{\forall t \geq 0}$ models the basicity of the concrete pH.

Inspired from the trends appropriate to these physical indicators, we propose: $\forall(\rho_1, \theta_1) > 0$,

$$\Delta\rho_1(\tau_1;\ \rho_1, \theta_1) \sim gamma\left(\alpha_{\rho_1}(\rho_1, \theta_1).\tau_1, \beta_{\rho_1}\right) \quad (1)$$

$$\Delta\theta_1(\tau_1;\ \rho_1, \theta_1, \Delta\rho_1) \sim gamma\left(\alpha_{\theta_1}(\rho_1, \theta_1, \Delta\rho_1).\tau_1, \beta_{\theta_1}\right) \quad (2)$$

with the suitable shape functions:

$$\alpha_{\rho_1}(\rho_1, \theta_1) = (a_3.\theta_1 + a_4).e^{\frac{-(\rho_1 - a_1)^2}{a_2}} \quad (3)$$

$$\alpha_{\theta_1}(\rho_1, \theta_1, \Delta\rho_1) = \left(a_6.\left(\rho_1 + \frac{\Delta\rho_1}{2}\right) + a_7\right).e^{-a_5.\theta_1} \quad (4)$$

For this study we consider the following parameters: $a_1 = 2.8$, $a_2 = 4.2$, $a_3 = 0.15$, $a_4 = 0.15$, $a_5 = 0.2$, $a_6 = 0.1$, $a_7 = 0.15$, $\beta_{\rho_1} = 0.2$ and $\beta_{\theta_1} = 0.2$.

3.2. Second phase

The crack initiation phase is characterized by:

- $(\rho_{2,t})_{\forall t \geq 0}$ represents the internal tensile stress (MPa).
- $(\theta_{2,t})_{\forall t \geq 0}$models the corrosion current density « i_{corr} » (µA/cm2).

From the trends appropriate to these physical indicators, we propose: $\forall(\rho_2, \theta_2) > 0$,

$$\Delta\theta_2(\tau_2\ ;\ \rho_2, \theta_2) \sim gamma\left(\alpha_{\theta_2}(\rho_2, \theta_2).\tau_2\ , \beta_{\theta_2}\right) \quad (5)$$

$$\Delta\rho_2(\tau_2\ ;\ \rho_2, \theta_2, \Delta\theta_2) \sim gamma\left(\alpha_{\rho_2}(\rho_2, \theta_2, \Delta\theta_2).\tau_2\ , \beta_{\rho_2}\right) \quad (6)$$

with the following shape functions:

$$\alpha_{\theta_2}(\rho_2, \theta_2) = (b_3.\rho_2 + b_4).e^{\frac{-(\theta_2 - b_1)^2}{b_2}} \quad (7)$$

$$\alpha_{\theta_2}(\rho_2, \theta_2, \Delta\theta_2) = \left(b_6.\left(\theta_2 + \frac{\Delta\theta_2}{2}\right) + b_7\right).e^{-b_5.\rho_2} \quad (8)$$

For this study we consider the following parameters: $b_1 = 3.1$, $b_2 = 3.2$, $b_3 = 1$, $b_4 = 0.15$, $b_5 = 0.25$, $a_6 = 0.05$, $a_7 = 1$, $\beta_{\rho_1} = 0.2$ and $\beta_{\theta_1} = 0.2$.

3.3. Third phase

The crack propagation phase is characterized by:

- $(\rho_{3,t})_{\forall t \geq 0}$ represents the width of the crack *a* (mm).
- $(\theta_{3,t})_{\forall t \geq 0}$models the corrosion current density i_{corr} (µA/cm2).

From the trends appropriate to these physical indicators, we propose: $\forall(\rho_3, \theta_3) > 0$,

$$\Delta\theta_3(\tau_3\ ;\ \rho_3, \theta_3) \sim gamma\left(\alpha_{\theta_3}(\rho_3, \theta_3).\tau_3\ , \beta_{\theta_3}\right) \quad (9)$$

$$\Delta\rho_3(\tau_3\ ;\ \rho_3, \theta_3, \Delta\theta_3) \sim gamma\left(\alpha_{\rho_3}(\rho_3, \theta_3, \Delta\theta_3).\tau_3\ , \beta_{\rho_3}\right) \quad (10)$$

with the suitable shape functions:

$$\alpha_{\theta_3}(\rho_3, \theta_3) = (c_3.\rho_3 + c_4).e^{\frac{-(\theta_3 - c_1)^2}{c_2}} \quad (11)$$

$$\alpha_{\theta_3}(\rho_3, \theta_3, \Delta\theta_3) = \left(c_6.\left(\theta_3 + \frac{\Delta\theta_3}{2}\right) + c_7\right).e^{-c_5.\rho_3} \quad (12)$$

For this study we consider the following parameters: $c_1 = 2.5$, $c_2 = 4$, $c_3 = 1$, $c_4 = 1.2$, $c_5 = 0.4$, $c_6 = 0.9$, $c_7 = 1$, $\beta_{\rho_3} = 0.2$ and $\beta_{\theta_3} = 0.2$.

4. MAINTENANCE ACTIONS

This section illustrates how maintenance actions can be represented by the meta-model. First, we will describe three maintenance actions adopted in the European EN 1504 applied to the case of chloride-induced corrosion. Then we explain the mathematical modeling of these maintenance actions.

4.1. Maintenance catalogue

Three maintenance techniques are chosen for this study: Cathodic protection (applied for the three phases), Chloride extraction (used in the diffusion phase) and Concrete replacement (available for the three phases). A technical guide on maintenance techniques can be found on http://durati.lnec.pt.

4.1.1. Cathodic protection

Cathodic protection (CP) is a technique used to control the corrosion by making it the cathode of an electrochemical cell. CP is an electrochemical technique installed permanently.

4.1.2. Chloride extraction

Chloride extraction (CE), sometimes called desalination, is an electrochemical process to remove chloride ions from a chloride contaminated concrete through ion migration. An anode embedded in an electrolyte medium is temporarily applied on the surface of the concrete.

4.1.3. Concrete replacement

Concrete replacement is used for restoring the original load-carrying capacity of damaged concrete or replacing a highly contaminated concrete.

In the case of chloride-induced corrosion, the concrete replacement can be applied at three different levels:

- CR1: it is a preventive repair strategy in which the structure is repaired before corrosion initiation. Chloride-contaminated concrete cover is repaired by removing few centimeters of material for slabs and beams (before concrete cover depth). Corroded bars are not replaced.
- CR2: it is a corrective repair strategy in which repair takes place after corrosion initiation but the loss of cross-sectional area of rebars is not significant. Cracked/chloride-contaminated concrete cover is repaired by removing about 6 cm of material for slabs and beams. Corroded bars are not replaced.
- CR3: it is a corrective repair strategy in which repair takes place after severe concrete cracking where the loss of cross-sectional area of rebars is significant. Cracked/chloride- contaminated concrete cover is repaired by removing about 6 cm of material for slabs and beams. Corroded bars are replaced.

4.2. Effect of a maintenance action on the meta-model

Maintenance actions can have different effects on the parameters: they can modify its speed by decelerating (i.e. CP) or accelerating the process (i.e. after CR, a small chloride content remains in the unremoved concrete inducing a chloride diffusion from the old material that can accelerate corrosion initiation). They can also change the level of degradation (i.e. CE removes given quantity of chlorides inside the concrete). Table 1 summarizes the possible effect of each maintenance action on the meta-model.

Table 1: Maintenance action effect on the meta-model

Maintenance actions	*1st Phase*		*2nd Phase*		*3rd Phase*	
	ρ_1	θ_1	ρ_2	θ_2	ρ_3	θ_3
CP	*d*	*d*	*d*	*d*	*d*	*d*
CE	$a*$	$a*$				
CR1	$a*$	$a*$				
CR2	$a*$	$a*$	$a*$	$a*$		
CR3	$a*$	$a*$	$a*$	$a*$	$a*$	$a*$
d: decelerates, a: accelerates, a: changes the degradation level to a lower one and a.*						

We propose to model the effect on maintenance action on the processes directly in the shape functions by introducing two new parameters $m_1$and$m_2$which can be defined respectively as the degradation acceleration factor and the effect of unremoved concrete after maintenance factor. Finally, for the average degradation rate, we multiply the shape function by a constant m_1 and for translation we introduce m_2 (Eq. 13, 14 and Figure 2).

On the other hand, a maintenance action can also modify the level of degradation. Having a state-dependent meta-model, with a markovian property, modifying the level does not require the modification of any of the model's functions. All it takes is to put the degradation at the appropriate level.

As a result, the shape function for an "S-shaped" trend is a "bell-shaped" function (Eq. 3, 7 and 11), after maintenance the shape function would be (Figure 2):

$$\alpha_S(\rho,\theta) = \boldsymbol{m_1} \times (a_3.\rho + a_4).e^{\frac{-((\theta-\boldsymbol{m}_2)-a_1)^2}{a_2}} \tag{13}$$

The shape function for an "L-shaped" trend is an "L-shaped" function (Eq. 4, 8 and 12), after 7 maintenance the shape function would be:

$$\alpha_L(\rho,\theta) = \boldsymbol{m_1} \times (a_3.\rho + a_4).e^{-a_1.(\theta-\boldsymbol{m_2})} \tag{14}$$

Maintenance techniques have been widely used and their effect on the physical process are rigorously studied. So the harder part in modelling the maintenance actions is to quantify m_1 and m_2. The estimation process is beyond the scope of this paper, but Maximum Likelihood Estimation procedure can be used towards this aim by using experimental data.

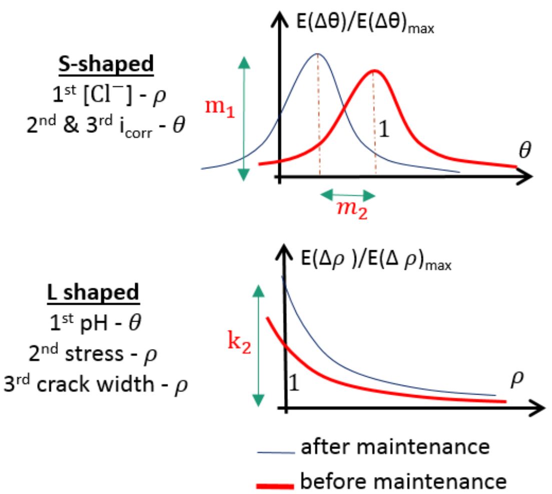

Figure 3. Mathematical model of a maintenance action on the shape functions

It is advantageous to be able to model a maintenance action using only two parameters per process instead of updating 9 parameters.

4.3. Performance indicators

The maintenance decision is chosen accordingly with the condition of the structure. In order to quantify the quality of a structure we need to choose an appropriate performance indicators.

Performance indicators can be classified under three categories (Frangopol *et al.*, 2007):

- *Condition indexes*: based on inspection and then classified in discrete states.
- *Safety indexes*: the structure can be either safe or unsafe.
- *Reliability indexes*.

In this study we consider a condition index (CI) based on the inspection values of the parameter of interest ρ. It is worth to be mentioned that a performance indicator can be a combination these categories.

We classify the structure into 10 discrete inspection-based states. Each phase is divided into 3 states starting from a CI=9 and going down CI=0. A CI=9 is associated with a low concentration of chloride (early phase of chloride diffusion) and ending in a CI=0 for a severely cracked structure (crack width of 3mm).

The range of each CI is defined by dividing the region between the horizontal axis and the threshold line into three un-even regions using square root intervals. The closer to the threshold a region gets, the narrowest it becomes (Figure 3).

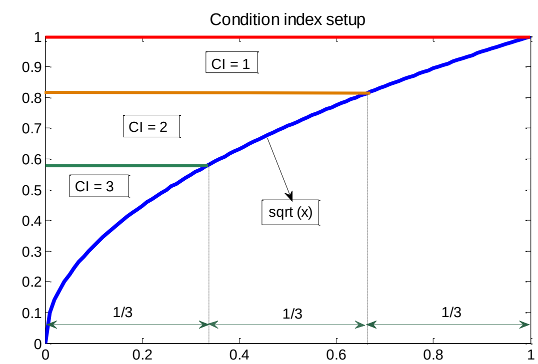

Figure 4. Condition states setup

5. APPLICATION OF THE MODEL IN A MAINTENANCE CONTEXT

This section illustrates the use of the meta-model for the assessment of life-cycle costs and condition indexes for different maintenance policies.

Preventive and corrective maintenance policies are considered. We will assume three lifetimes of 50, 75 and 100 years. The inter-inspection time is fixed at a 5 years. Inspections are considered as perfect.

This example aims at assessing the maintenance actions and costs for a target CI. We evaluate maintenance costs and condition index by carrying out Monte Carlo stochastic simulations under the under the Matlab® environment. The outputs of simulations are simulated inspection data and the history of the structures (e.g., loss of steel). The decision of a repair after inspection is based on these outputs and depends on both maintenance policy and extent of damage.

For this example we consider only the concrete replacement repair methods: CR1, CR2 and CR3. This choice was made since we dispose of real costs for the repair of a marine harbor (Srifi, 2012). For each policy, 1000 simulations are carried out to determine the costs (inspection, maintenance and total) and the CI of the structure. Total cost for each policy are assessed at present time without considering a discounting factor. Since we focus on existing structures, construction and salvage cost are not included in the analysis.

Table 2: Costs of maintenance and inspections

Phase	*CR1*	*CR2*	*CR3*
Maintenance cost (€/m²)	*263.2*	*323.0*	*353.4*
Inspection (€/m²)	*25*	*25*	*10*

5.1. Preventive Maintenance

The preventive maintenance (PM) policy aims at repairing the structure for a target CI = 7 (end of the 1st phase). The objective of this policy is to prevent the initiation of corrosion.

However, having a 5 years inter-inspection interval, it is possible to miss an inspection for a CI = 7. In this case, we have to inspect for (ρ_2, θ_2) to make sure that the structure is in the 2nd phase, and the corrosion have already started. In that case, a CR2 is required instead of the originally planned CR1 generating more costs, hereafter called over cost.

From a practical point of view, if corrosion starts many times during the structural lifetime, the loss of steel might be significant enough to consider a CR3 (concrete replacement with replacement of reinforcement steel) adding more over-cost.

Table 3 gives the costs and the CI for three different life times of a structure maintained through a preventive policy.

The Condition Index in the tables is the mean of all the inspections' condition indexes.

The over costs represents about the 11% of the total cost. Clearly the inter-inspection time have an impact on this value. Also, the relatively small size of the CI=7 zone compared to the two zones of the 1st phase (9 and 8) is another factor. If the zone 7 was bigger, it is more certain to perform a maintenance for the aimed CI rather than skipping to the 2nd phase, but also we can trigger an early maintenance.

Table 3: Costs and Condition index for a preventive maintenance policy

Lifetime (years)	*50*	*75*	*100*
Inspections (€/m²)	*318.2*	*465.4*	*619.9*
Maintenance (€/m²)	*864.3*	*1299.5*	*1804.2*
Total cost (€/m²)	***1182.5***	***1764.9***	***2424.1***
Annual cost (€/m²)	*23.6*	*23.5*	*24.2*
Over cost (€/m²)	*146.6*	*221.8*	*321.9*
Condition Index	***8.22***	***8.2***	***8.15***

5.2. Corrective Maintenance

The corrective maintenance policy aims at repairing the structure after sever cracking (CI=0). Table 4 provides the have the costs and the CI for three lifetimes.

Table 4: Costs and Condition indexes for a corrective maintenance policy

Lifetime (years)	*50*	*75*	*100*
Inspections (€/m²)	*322*	*468.7*	*620.6*
Maintenance (€/m²)	*352*	*628.3*	*783.1*
Total cost (€/m²)	***674***	***1097***	***1403.8***
Annual cost (€/m²)	*13.5*	*14.6*	*14*
Condition Index	***6.38***	***5.85***	***5.86***

5.3. Comparison

From Table 3 and Table 4 we can see that the corrective maintenance compromises the CI (~6) but reduces the costs (almost by half). In contrast, the preventive maintenance policy maintains the structure at a higher level of performance (CI>8) with a larger maintenance cost.

It is essential to point out that the ultimate objective of this study is to show the applicability of the meta-model in a maintenance optimization context, which was done here.

6. CONCLUSIONS AND PERSPECTIVES

Degradation models are an essential tool to predict the evolution in time of structural performance. These models are then paramount for planning and optimizing maintenance and management policies.

This work proposed a new degradation meta-model that accounts for the above mentioned purposes. It can be seen as an intermediate between physical models (that can be complex to apply in a reliability context), and pure probabilistic models (that fail to reflect the physical degradation and therefore suffer from lack of acceptability by the Civil engineering community). Moreover, the construction and calibration of the model are done via NDT data.

This paper also illustrated the applicability of the meta-model in a maintenance context.

Future works will consider real data for the calibration and will focus on maintenance optimization. Concerning maintenance optimization, the next step would be to set up and define the multi-objective optimization problem.

7. ACKNOWLEDGEMENT

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